AD-A103 171   PRINCETON UNIV NJ DEPT OF CIVIL AND GEOLOGICAL ENGIN--ETC F/G 20/11
NONLOCAL EFFECTS IN BRITTLE CRACK PROPAGATION.(U)
JUL 81 N ARI, A C ERINGEN
UNCLASSIFIED 81-SM-24   N00014-76-C-0240

1 of 1
AD A
103171

END
DATE
FILMED
9-81
DTIC

NONLOCAL EFFECTS
IN BRITTLE CRACK PROPAGATION

N. Ari and A.C. Eringen
Princeton University

Technical Report No. 56
Civil Engng. Res. Rep. No. 81-SM-24

Research Sponsored by the
OFFICE OF NAVAL RESEARCH
under
Contract N00014-76-C-0240 Mod 4
Task No. NR 064-410

July 1981

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER PRINCETON UNIVERSITY TECHNICAL REPORT #56	2. GOVT ACCESSION NO. AD-A103 177	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NONLOCAL EFFECTS IN BRITTLE CRACK PROPAGATION		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER 81-SM-24
7. AUTHOR(s) Nasit Ari and A.C. Eringen		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C02040 Mod. 4
9. PERFORMING ORGANIZATION NAME AND ADDRESS Princeton University Princeton, NJ 08544		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 064-410
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH (Code 471) Arlington, VA 22217		12. REPORT DATE July 1981
		13. NUMBER OF PAGES 29
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

crack propagation
dynamic fracture
nonlocal fracture

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Stress distribution near the tip of a constant velocity crack is determined by means of nonlocal theory of elasticity. The maximum stress is finite and it allows one to utilize the maximum stress hypothesis to determine the terminal velocity of running cracks.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

NONLOCAL EFFECTS IN BRITTLE CRACK PROPAGATION*

N. Ari and A.C. Eringen
Princeton University
Princeton, NJ 08544

ABSTRACT

Stress distribution near the tip of a constant velocity crack is determined by means of nonlocal theory of elasticity. The maximum stress is finite and it allows one to utilize the maximum stress hypothesis to determine the terminal velocity of running cracks.

1. INTRODUCTION

One of the fundamental mechanisms of dynamic rupture phenomena is that of crack propagation. The angular distribution and the maxima of the crack tip stresses vary strongly with the crack velocity and the dynamic effects become important. One has to solve an elastodynamic problem to obtain these dynamic stresses so that at least a qualitative understanding of the crack motion can be understood.

The present work investigates stress distribution near the tip of a moving crack in a brittle elastic solid by means of the recently developed theory of nonlocal elasticity [1].

---

*Supported by the Office of Naval Research.

The dynamic crack problems are rather difficult to solve without some simplifying assumptions. Most common among them are the constant crack tip velocity and/or the loading conditions which allow self-similar crack motions. As such, these problems represent a highly contrived picture of the actual rupture phenomenon. Nevertheless, these models prove to be useful in discussing the velocity dependence of the crack tip stresses.

One of the earliest and simplest models of a moving crack is the Yoffe's model [2]. A straight line crack of fixed length 2ℓ moves with a constant velocity in an infinite plate subject to uniform tension perpendicular to the crack (Fig. 1). Below, we present a solution of Yoffe's problem within the context of nonlocal elasticity.

The classical elasticity solution of Yoffe's problem as well as other classical treatments of uniformly moving cracks (3-5) have provided certain insights into dynamical aspects of brittle fracture and inspired subsequent research. However, these studies have remained inadequate in explaining a number of important features of dynamic crack propagation. To this end, the following points are worth noting:

(i) The classical elasticity solution yield the familiar square root stress singularities at crack tips. In order to circumvent the difficulties related to infinite stresses, the dynamic fracture criteria are based on energy considerations (energy release rate). However, the dynamic failure is localized at the crack tip and it depends on the critical stress levels. A maximum stress criterion is therefore more appropriate.

(ii) The dynamic stress intensity factor in Yoffe's solution

$$(1.1) \qquad K = \lim_{r \to 0} [(2\pi r)^{\frac{1}{2}} t_{yy}(r,\theta=0)]$$

turned out to be independent of crack tip velocity and equal to its static value. This result is contrary to experimental observations that K should vary with the crack speed.

(iii) The analytic expressions of the classical solutions imply that the Rayleigh wave velocity c_R is a natural upper bound for the crack tip velocity. However, for all materials observed, the terminal velocity V_t remained considerably less than c_R [6]. Since Mott [7] first addressed the problem of a modified Griffith criterion for a moving crack, the determination of the limiting velocity V_t has been an open problem. Operating on the premise of a constant surface energy γ_0, Roberts and Wells arrived at a formula expressing the terminal velocity [8]. However, experimental evidence indicates that the surface energy varies with the crack velocity, i.e. $\gamma = \gamma(V)$. Therefore, Freund [9] considers any agreement of their theoretical terminal velocity with the experimental one rather accidental. An additional difficulty in dealing with surface energy concept is its experimental determination and accuracy.

(iv) A more involved fracture phenomenon is that of crack bifurcation -- a rapidly moving crack suddenly branches into two new cracks. The precise physical source of this mechanism is not known. Neither is it settled as to which criterion is better suited to determine the

prebranching velocity and the bifurcation angles once the solution for the dynamic problem is given [10].

The motivation for the present work is derived from the foregoing considerations. In order to develop controlled fracture processes and crack arrest mechanisms, the need for addressing the above questions is clear. Encouraged by the results of the static nonlocal crack problems [11-13], we will pursue the solution of the nonlocal Yoffe problem. The nonlocal theories incorporate the long range interactions and the microstructure dependence intrinsically. Hence, they provide a more realistic framework for the treatment of crack problems.

Finally, the formulation of a mixed boundary value problem in nonlocal elastodynamics is in itself important. Difficulties related to mixed boundary conditions have been considered in our earlier work [12,14]. In the present work, in addition, we discuss a modification of the field equations due to the dynamic effects. Throughout the work, emphasis will be on how the nonlocality modifies the dynamic crack tip stresses.

Section 2 formulates the nonlocal boundary value problem. In Section 3, the ensuing dual integral equations are solved. The velocity dependence of the crack tip stresses is given in Section 4. Section 5 compares the nonlocal results with the classical ones and discusses their relevance to dynamic fracture criteria. The quantitative results are gratifying.

2. NONLOCAL FORMULATION

In nonlocal elasticity for isotropic solids, the stress constitutive equations are given by [1].

$$(2.1) \qquad t_{k\ell} = \iint_R [\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)\, e_{rr}(\underset{\sim}{x}')\delta_{k\ell} + 2\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)e_{k\ell}(\underset{\sim}{x}')]da(\underset{\sim}{x}')$$

where the integral is over the two-dimensional plane region $R(x',y')$ and $e_{k\ell}$ is the strain tensor which is related to the displacement vector u_k by

$$(2.2) \qquad e_{k\ell} = \frac{1}{2}(u_{k,\ell} + u_{\ell,k})$$

Here, and throughout an index following a comma represent gradient, e.g.

$$u_{k,\ell} = \partial u_k/\partial x_\ell$$

The nonlocal kernels $\lambda'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ and $\mu'(|\underset{\sim}{x}'-\underset{\sim}{x}|)$ represent the influence of the neighboring strains at $\underset{\sim}{x}'$ on the nonlocal stress at a reference point $\underset{\sim}{x}$. They are usually determined by requiring that the nonlocal field equations yield identical dispersion relations to those derived from atomic lattice dynamics. In this way, in [12], we introduced a two-dimensional kernel

$$(2.3) \qquad \lambda'/\lambda = \mu'/\mu = \alpha(|\underset{\sim}{x}'-\underset{\sim}{x}|) = \frac{1}{2\pi}\beta^2 K_0\{\beta[(x'-x)^2 + (y'-y)^2]^{\frac{1}{2}}\}$$

where K_0 is the zeroth-order modified Bessel function and β is the nonlocality parameter. In the local limit $\beta \to \infty$, (2.3) reverts into a Dirac-delta functional and (2.1) yields the classical Hooke's law. In addition, (2.3) has the convenient property that

$$(1 - \varepsilon^2\nabla^2)\,\alpha(x,y) = \delta(x,y) \tag{2.4}$$

where $\varepsilon = 1/\beta$ and δ is the Dirac-delta functional.

For nonlocal elasticity (when the effects of nonlocal residuals and the body forces are excluded) the integral balance equations of linear momentum yield the equations of motion

$$t_{k\ell,k} = \rho\ddot{u}_\ell \tag{2.5}$$

where a superposed dot represents the material time derivative. Due to the integral form of constitutive equations, (2.5) is of integro-differential character. For such systems, it is very difficult to establish existence and uniqueness theorems and special care is needed in the formulation of mixed boundary value problems. In addition, a closer look at the field equations signals certain difficulties for cases where the displacement fields are discontinuous such as in bodies with cracks. For example, in (2.5), when $\ddot{u}_\ell$ term is discontinuous, the $t_{k\ell,k}$ term is very likely not to be discontinuous, since the discontinuities in the displacement fields will be smoothed out by the integral operator in (2.1).

With the appearance of the crack, some interatomic bonds are eliminated and inhomogeneities arise within a narrow boundary layer near the crack surfaces. The nonlocal body forces become important and the use of nonhomogeneous kernels are required. Since, however, the inhomogeneities are confined to a narrow domain, we can approximate the problem with a homogeneous kernel, provided the compatibility of the crack discontinuities with the nonlocal field equations are ensured.

To this end, a modification of the field equations can be obtained by utilizing the relation (2.4), i,e,

$$(2.6) \qquad (1-\varepsilon^2\nabla^2)\, t_{k\ell,k} = \sigma_{k\ell,k} = (1-\varepsilon^2\nabla^2)\, \rho \ddot{u}_\ell$$

where

$$\sigma_{k\ell} = \lambda e_{rr}\, \delta_{k\ell} + 2\mu\, e_{k\ell}$$

is the local (classical) stress tensor. (2.6) together with the appropriate side conditions constitute a sufficient solution for the integral balance equations. The modified equations of motion are singularly perturbed differential equations and they are simpler than the integro-differential system given by (2.5). (2.6) can be derived alternatively from integral balance equations by imposing uniform continuity requirements on nonlocal stresses and displacements [15].

In the two-dimensional formulation, we assume that the displacement fields can be derived from wave potentials $\phi(X,Y,t)$ and $\psi(X,Y,t)$, i.e.

$$(2.7) \qquad \begin{aligned} u &= \partial\phi/\partial X + \partial\psi/\partial Y \\ v &= \partial\phi/\partial Y - \partial\psi/\partial X \end{aligned}$$

Substituting (2.7) into (2.6), we obtain two scalar wave equations

$$(2.8) \qquad \begin{aligned} (1 - \varepsilon^2\nabla^2)\, \ddot{\phi} &= c_1^2\nabla^2\, \phi \\ (1 - \varepsilon^2\nabla^2)\, \ddot{\psi} &= c_2^2\nabla^2\, \psi \end{aligned}$$

where

$$c_1 = [(\lambda + 2\mu)/\rho]^{\frac{1}{2}}, \qquad c_2 = (\mu/\rho)^{\frac{1}{2}}$$

which must be solved to determine the displacement field. In the case of the constant velocity crack problem, it is more convenient to introduce a moving coordinate system (Fig. 1),

$$x = X - Vt \qquad y = Y \tag{2.9}$$

where V is the crack tip velocity. We assume that in the new coordinate system, the wave potentials become independent of time. Then the field equations (2.8) reduce to

$$\left\{\varepsilon^2 V^2 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + c_1^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - V^2 \frac{\partial^2}{\partial x^2}\right\} \phi(x,y) = 0$$
$$\left\{\varepsilon^2 V^2 \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + c_2^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - V^2 \frac{\partial^2}{\partial x^2}\right\} \psi(x,y) = 0 \tag{2.10}$$

By means of the Fourier transform in the x-direction, we obtain the general solution of (2.10) satisfying the conditions $\phi,\psi \to 0$ as $(x^2+y^2)^{1/2} \to \infty$

$$\begin{aligned} \phi(k,y) &= A(k)\, e^{-\gamma_1(k)y} \\ \psi(k,y) &= B(k)\, e^{-\gamma_2(k)y} \end{aligned} \qquad y > 0 \tag{2.11}$$

where k is the transform variable and

$$\gamma_i^2(k) = [(c_i^2 - V^2 - \varepsilon^2 k^2 V^2)/(c_i^2 - \varepsilon^2 k^2 V^2)]k^2 \;; \qquad i=1,2 \tag{2.12}$$

The inversion contour of (2.11) requires proper branch cuts so as to ensure that

$$(2.13) \qquad \mathrm{Re}\, \gamma_i(k) \geq 0$$

The boundary conditions for moving crack in the new coordinate system is equivalent to the static case. Therefore, the self-consistent nonlocal boundary conditions are given by, [12] ,

$$(2.14) \qquad \begin{aligned} \sigma_{yy}(x,0) &= -t_0 && |x| < \ell \\ v(x,0) &= 0 && |x| > \ell \\ \sigma_{xy}(x,0) &= 0 && \forall\, x \\ u,v \to 0 & \quad \text{as} \quad (x^2+y^2)^{\frac{1}{2}} \to \infty \end{aligned}$$

(2.14) and (2.11) lead to a set of dual integral equations whose solution will determine $A(k)$ and $B(k)$.

3. SOLUTION OF THE DUAL INTEGRAL EQUATIONS

Substituting (2.11) and (2.7) into (2.14), we obtain

$$(3.1)\quad \sigma_{yy}(x,0) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \{[(\lambda+2\mu)\ \gamma_1^2 - \lambda k^2]\ A(k) - 2\mu\ ik\ \gamma_2\ B(k)\}\ e^{-kx}\, dk = -\,t_0\ ; \qquad |x| < \ell$$

$$(3.2)\quad v(x,0) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} [-\gamma_1\ A(k) + ik\ B(k)]\ e^{-ikx}\, dk = 0\ ; \qquad |x| \geq \ell$$

$$(3.3)\quad \sigma_{xy}(x,0) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \mu\{2ik\ \gamma_1 A(k) + (\gamma_2^2 + k^2)\ B(k)\ \ e^{-ikx}\, dk = 0 \ ; \quad \forall x\ .$$

An exact solution to the system is yet to be found. *The problem is further* complicated by the fact that the $\gamma_i(k)$ of (2.12) assumes complex values along the real axis between their respective branch points.

In the static crack problems, it has been shown that the classical solution constitutes a reasonable approximation to the nonlocal one [11,12]. The only difference between the classical and nonlocal versions of the system (3.1) - (3.3) is the dependence of $\gamma_i(k;\varepsilon)$ to ε, where $\gamma_i(k, \varepsilon{=}0) = s_i k$

$$(3.4)\qquad s_i^2 = 1 - (V/C_i)^2\ .$$

In the limit as $\varepsilon \to 0$, we recover the classical solution. The behaviour of integrands in (3.1) - (3.3) for large k is similar to the classical case,

therefore, it is expected that the limiting process, as $\varepsilon \to 0$, shall be uniform. Hence as an approximation, one might use the classical solution provided that the complex values of $\gamma_i(k)$ are taken into account. To this end, we choose a modified form of the classical solution such that $\gamma_i(k)\ A_N(k)$ and $\gamma_i(k)\ B_N(k)$ remain always real valued and numerically equal to the absolute values of $\gamma_i A_C$ and $\gamma_i B_C$ respectively. These conditions will be satisfied if the nonlocal solutions for A_N and B_N are expressed as

$$A_N(k) = P_1(k) + i\ P_2(k) \tag{3.5}$$

$$B_N(k) = Q_1(k) + i\ Q_2(k) \quad , \tag{3.6}$$

where

$$P_1(k) = A_C(k) \qquad Q_1(k) = B_C(k)\ ; \quad \forall k \tag{3.7}$$

$$P_2(k) = -\ A_C(k) \qquad Q_2(k) = -\ B_C(k); \quad k\ R \tag{3.8}$$

and R correspond to those regions of the real k-line on which $\gamma_1(k)$ or $\gamma_2(k)$ are not real valued.

The classical solutions are given by [16]

$$A_C(k) = (1 + s_2^2)\ D_0 J_1(k\ell)/k^2 \tag{3.9}$$

$$B_C(k) = -\ 2\ s_1 i\ D_0 J_1(k\ell)/k^2 \tag{3.10}$$

where

$$D_0 = (\frac{\pi}{2})^{\frac{1}{2}} \; t_0 \quad \ell (V/C_2)^2 \; [4s_1 s_2 - (1-s_2^2)^2]^{-1}$$

It can be shown by substitution that the nonlocal boundary conditions are uniformly approximated by (3.5) - (3.6) to the order of $0\ (\varepsilon^{\frac{1}{2}})$ [15].

In the present work, we are primarily interested in the normal stress along the crack line, thus in the next section, we utilize this modified classical solution to express the normal stress along the crack line near the tip.

4. NONLOCAL STRESSES

The constitutive equations for the stress field are given by (2.1). Using the standard integration formulas and substituting (3.5)-(3.10) into (2.1), we obtain for the normal stress along the crack line

$$t_{yy} = k_1 [4\, s_1 t_B - 2\,(1+s_2^2)\; t_A] \tag{4.1}$$

where

$$\{t_A, t_B\} = \int_0^\infty J_1(k) \cos k\,\xi \;\{\alpha_A(\varepsilon k),\; \alpha_B(\varepsilon k)\}\; dk \tag{4.2}$$

$$\alpha_A(\varepsilon k) = \frac{(C_1^2 - \eta^2 V^2 - \varepsilon^2 k^2 V^2)}{C_1^2} \left[1 - \frac{\varepsilon k}{(1+\varepsilon^2 k^2)^{\frac{1}{2}}} \left(\left| \frac{(C_1^2 - V^2 - \varepsilon^2 k^2 V^2}{(C_1^2 - \varepsilon^2 k^2 V^2)} \right|\right)\right] \tag{4.3}$$

$$\alpha_B(\varepsilon k) = \left\{\left(\left|\frac{C_2^2 - V^2 - \varepsilon^2 k^2 V^2}{C_2^2 - \varepsilon^2 k^2 V^2}\right|\right) - \frac{\varepsilon k}{(1+\varepsilon^2 k^2)^{\frac{1}{2}}} \frac{C_2^2 - V^2 - \varepsilon^2 k^2 V^2}{C_2^2 - \varepsilon^2 k^2 V^2}\right\} \frac{C_2^2 - \varepsilon^2 k^2 V^2}{C_2^2} \tag{4.4}$$

and

$$k_1 = [4\, s_1 s_2 - (1 + s_2^2)^2]^{-1} \quad t_0 \quad \ell^{\frac{1}{2}}/2\pi^{\frac{1}{2}}$$

$$(4.5) \qquad \eta^2 = (\lambda+2\mu)/2\mu$$

$$\xi = x/\ell \; ; \qquad \varepsilon = 1/\beta\ell$$

Near the crack tip, stresses are given in terms of generalized hypergeometric functions which are difficult to visualize in graphic terms (see Appendix 1). In the present work, we are most interested in the velocity dependence of the crack tip stresses. Therefore we present the numerical values of $t_{yy}(1,0;\,V)$ in Table 1.

An alternative method of solution to the system (3.1) - 3.3) is the reduction of the dual integral equations into an equivalent Fredholm integral equation of the second kind. The Fredholm integral was then solved numerically. The crack tip stresses so obtained are comparable to the analytic results as shown in Table 2.

5. DISCUSSION

The following results are worth noting:

(i) The nonlocal stresses at the crack tip are finite. These results will enable us to extend Eringen's fracture criterion [17] to dynamic rupture phenomena. In the limit as the crack velocity $V \to 0$, we recover the static nonlocal crack results (see Table 1).

(ii) The local analysis of the steady-state crack problems yielded stress intensity factors which are independent of the crack velocity and equal to their static values (Yoffe [2] Craggs [3]). These results are clearly undesirable in that they contradict the experimental results which show that the crack tip velocity depends on applied loads.

The nonlocal analysis on the other hand, provided a crack tip stress behaviour which is compatible with experiments. The substantial difference between the local and nonlocal results can be observed in Figure 2.

(iii) The nonlocal results can be employed further to determine the terminal velocity of propagating cracks. The uniformly moving crack is in a state of dynamic equilibrium. As the crack propagates first, the interatomic bonds at the tip break. Then as the stress around the crack tip relaxes, the next bond is overloaded by the sweeping stress wave. There will be a sudden increase in the stress level above the ultimate stress one atomic distance away from the tip.

At the same time, from Fig. 2, we observe that the crack tip stress decreases with increasing velocity. In the dynamic equilibrium, two changes in the stress level compensate for each other and the crack reaches a uniform propagation velocity.

The stress relaxation at the crack tip is a complex phenomenon. Therefore, it is difficult to calculate the stress overshoot precisely. However, using a harmonic oscillator model, we may assume that it is roughly equal to the difference between the crack tip stress and the stress level one atomic distance away from the tip. Consequently, we propose that the terminal velocity be determined by

$$(5.1) \qquad \frac{t_{yy}(1 + a;\, 0)}{t_{yy}(1\,;\, 0)} \sim \frac{t_{yy}(1;\, V_t)}{t_{yy}(1;\, 0)}$$

$t_{yy}(1+a)/t_{yy}(1)$ is given in [12, Table 1] for the static crack problem. $t_{yy}(v)/t_{yy}(0)$ is plotted on Fig. 2. On Fig. 3, both curves are drawn together -- their crossing point yields the value of the terminal velocity as

$$(5.2) \qquad V_t = \rho c_2 \qquad \rho \sim (0.5 - 0.6)$$

Note that, in the case one is able to solve the stress relaxation problem at the crack tip, $[t_{yy}(1+a)/t_{yy}(1)]$ in Fig. 3, may not be a straight line any more (i.e. it will be velocity dependent). In that case, the terminal velocity estimates will be slightly modified.

(iv) By formal extension of Eringen's fracture criterion to dynamic phenomena, we can predict qualitatively the variation of the surface energy with the propagation velocity. The Griffith definition of surface energy can be formally extended to the velocity dependent case by

$$t_C^2\, a = \frac{8\mu}{(1-\nu)}\, C(V)^2\, \gamma(V) \quad , \tag{5.3}$$

where $\gamma(V)$ is the surface energy, μ,ν are the standard material moduli, t_C is the cohesive stress, a is the inter-atomic distance and $C(V)$ is defined by

$$t_{yy}(l,0)/t_0 = C(V)\,(2\ell/a)^{1/2} \quad . \tag{5.4}$$

By assuming the cohesive strength, a material property and therefore constant, from (5.3) we can deduce that

$$\frac{\gamma(V)}{\gamma(0)} = \left[\frac{C(0)}{C(V)}\right]^2 \tag{5.5}$$

From (5.3) and Table 1, we can deduce that the surface energy increases with increasing crack velocity. Although there is considerable scatter in the experimental values of dynamic surface energies, the nonlocal results are in agreement with their general tendency to increase with crack speed [9].

(v) If one wishes to establish a link with classical fracture theories, the following is interesting to mention. A typical fracture toughness versus crack velocity relationship is shown in Figure 4 [18]. The general character of this relationship is supported by physical considerations (Erdogan [19], Freund [20]), as well as by experiments on structural mild steel [21, 22] and on glassy polymers [23]. On Figure 4, we also plotted the ratio $t_{yy_{max}}(V)/t_{yy_{max}}(0)$.

Figure 4 indicates that in the lower velocity ranges, the fracture toughness decreases faster than the maximums tress with increasing velocity. Thus when

$$t_{yy_{max}}(V) > t_{yy_{critical}}(V) \quad , \tag{5.6}$$

the crack will accelerate. According to Eringen's fracture criterion, a dynamic equilibrium will be established when

$$t_{yy_{max}}(V_t) = t_{yy_{critical}}(V_t) \quad . \tag{5.6}$$

We can therefore determine the terminal velocity, assuming that it is defined by this dynamic equilibrium. From Figure 4, we observe that the terminal velocity will be on the order of

$$V_t \sim 0.55\ C_2 \ . \tag{5.7}$$

The important feature of this result is in that it predicts a terminal velocity well below the Rayleigh wave speed,[1] which is consistent with experimental observations and some numerical simulation studies [24]. Furthermore, it is obtained by a simple, natural fracture criterion, without extraneous arguments. The present approach, however, should be considered with some caution, since the fracture toughness measurements include some global effects such as viscoelasticity and plasticity. Therefore, we prefer the microstructural considerations discussed in (iii), to predict the terminal velocity.

(vi) The explanation of crack bifurcation phenomena is much more difficult. Experimental observations suggest that [25]

(1) $$\sigma_f \, C_b^{1/2} = \text{constant}$$

where σ_f is the fracture stress and C_b is thelength of the crack at the moment of branching.

(2) "Crack branching is not a spontaneous event"... but is the eventual outcome of cumulative process of advance cracking.

These characteristics of crack branching suggest that it is not likely to explain crack bifurcation solely in terms of a critical branching velocity. One has to consider the fracture energies of the involved materials (i.e. glasses do have low but tool steels do have high fracture energies). In addition, the accelerating phase of the crack propagation has to be taken into account.

[1] For example, Broberg's local analysis [4] predicts $V_t = V_R$.

Our present analysis does not include considerations about advance cracking and acceleration effects. Hence we do not attempt to explain crack branching on the basis of the present result. Nevertheless, from the foregoing conclusions, it is clear that the nonlocal dynamic results are in the right direction.

TABLE IV.1

VARIATION OF CRACK TIP STRESS
WITH THE CRACK TIP VELOCITY

(from 4.3, $\lambda = \mu$)

V/C_2	$t_{yy}(1,0;V)/t_{yy}(1,0;\ V=0)$
0.1	0.96
0.2	0.93
0.3	0.89
0.4	0.84
0.5	0.77
0.6	0.68
0.7	0.55
0.8	0.27

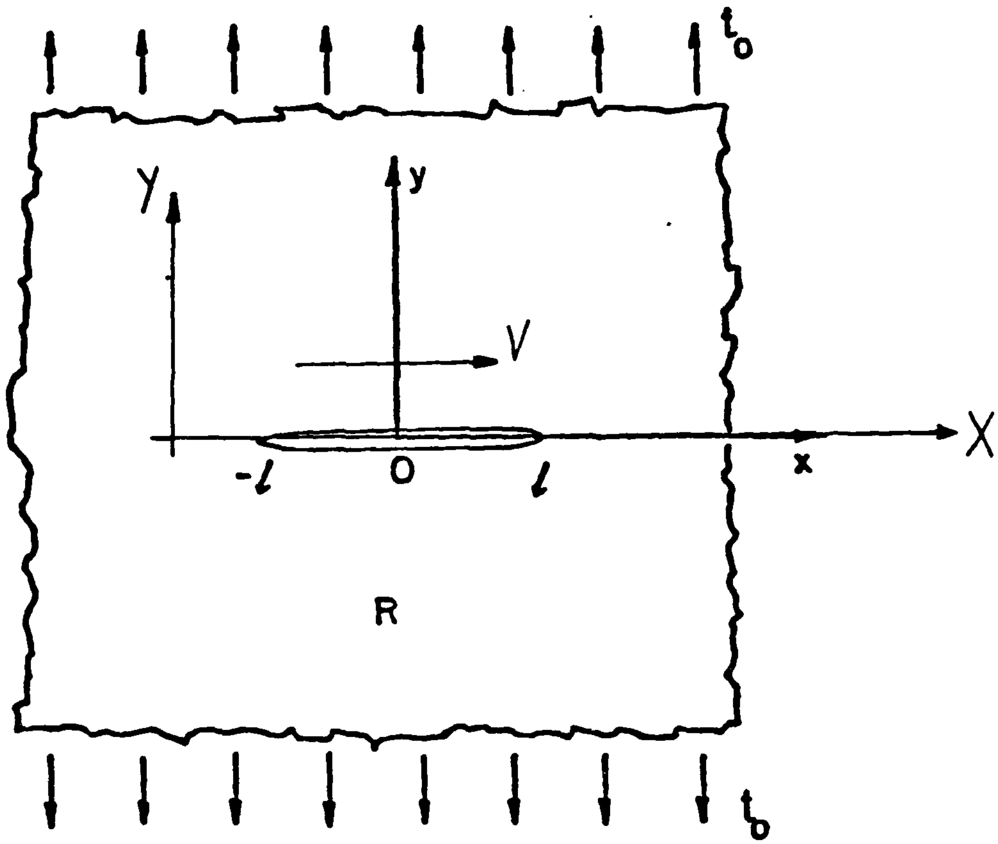

FIGURE 1

PLATE WITH MOVING LINE CRACK

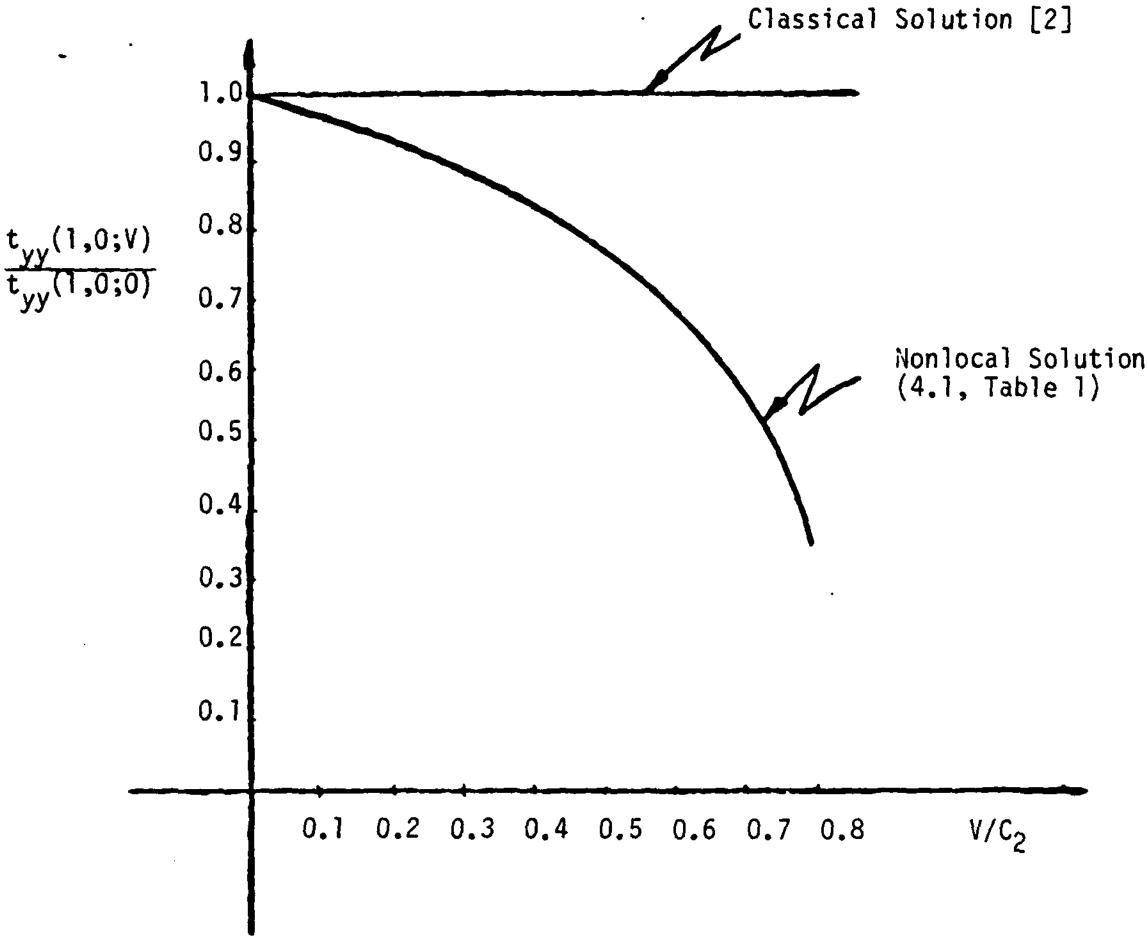

FIGURE 2

COMPARISON OF LOCAL AND NONLOCAL
CRACK TIP STRESSES

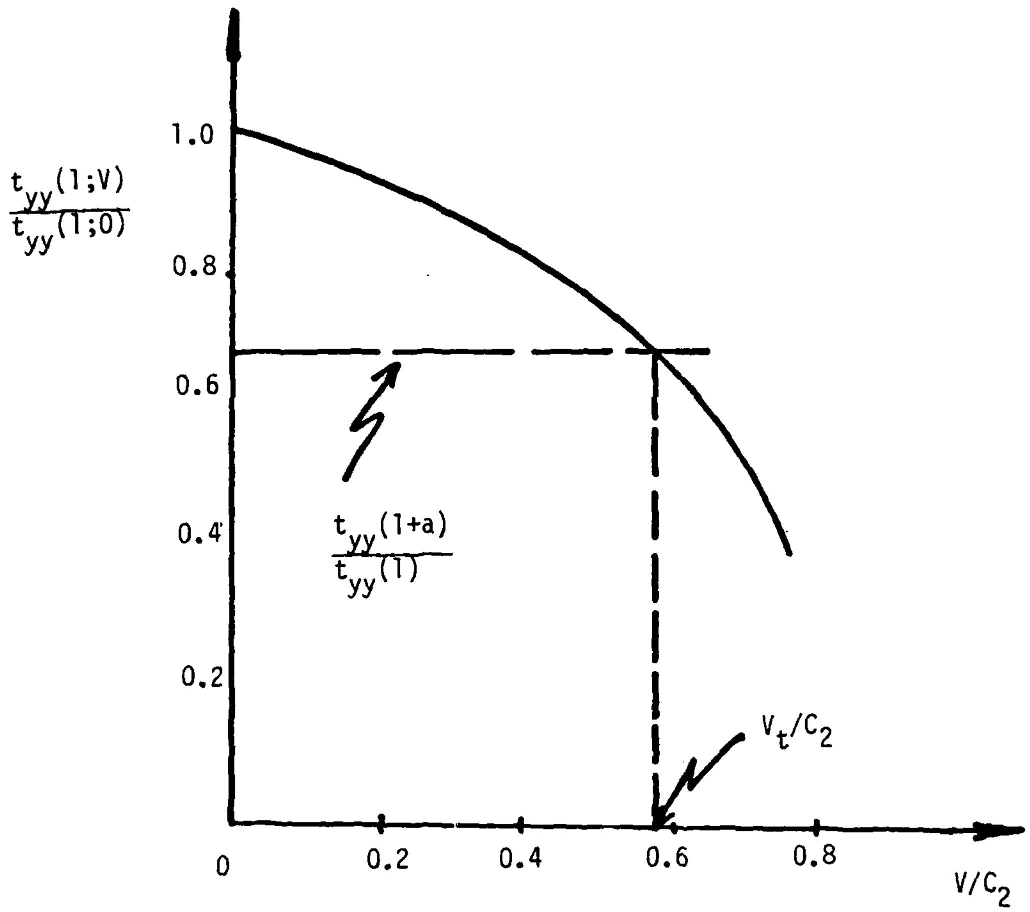

FIGURE 3

TERMINAL VELOCITY
(Nonlocal Determination)

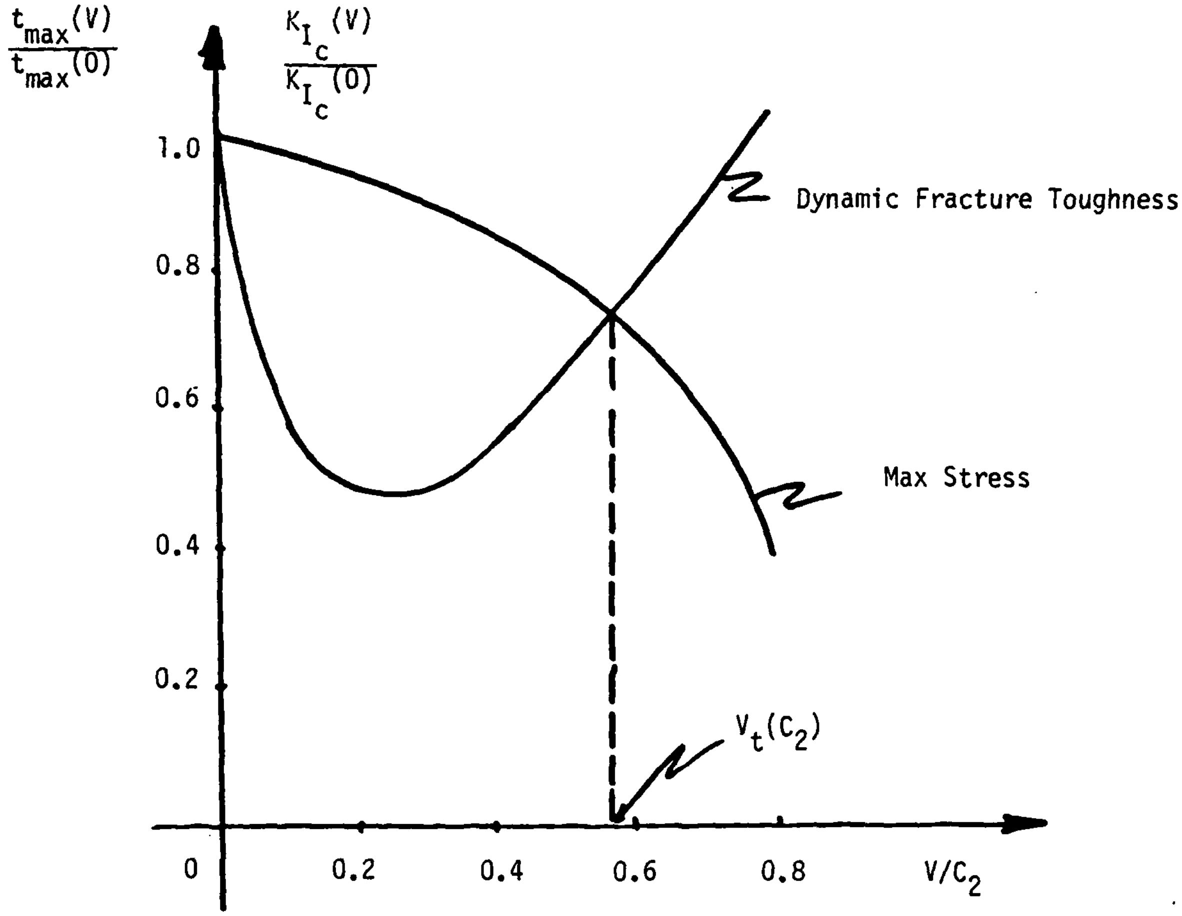

FIGURE 4

TERMINAL VELOCITY DETERMINATION
BY FRACTURE TOUGHNESS DATA

APPENDIX 1

The evaluation of the stress tip integrals can be facilitated by the use of the asymptotic expansion of Bessel functions, i.e.

(A.1) $$J_1(k) \sim k^{-\frac{1}{2}} \quad \cos(k - 3\pi/4) + O(k^{-3/2}) \; ; \qquad |k| \to \infty$$

Substituting (A.1) into (4.2), we obtain the following results as $\xi \to 1^+$

(A.2) $$t_A = [(C_1^2 - n^2V^2)/C_1^2]\,(t_a + t_c + t_e) - (t_b + t_d + t_c)$$

(A.3) $$t_B = t_g + t_i + t_k$$

where

(A.4) $$t_a = 2\, s_1^{\frac{1}{2}} M_1^{\frac{1}{2}} - 0.5\, s_1^{5/2} M_1^{3/2} \frac{\pi^{1/2}}{2^{5/4}} \frac{\Gamma(3/2)\,\Gamma(3/4)}{\Gamma(9/8)\,\Gamma(13/8)}$$

$$_3F_2(1/2,\ 3/2,\ 3/4;\ 13/8,\ 9/8;\ q_a)$$

(A.5) $$t_b = 0.4\, s_1^{5/2} M_1^{1/2} - 0.5\, s_1^{9/2} M_1^{3/2} \frac{\pi^{1/2}}{2^{9/4}} \frac{\Gamma(3/2)\,\Gamma(7/4)}{\Gamma(13/8)\,\Gamma(17/8)}$$

$$_3F_2\,(1/2,\ 3/2,\ 7/4;\ 17/8,\ 13/8;\ q_a)$$

$$q_a = -s_1^4 M_1^2/4$$

(2.6) $$t_c = 2(1-s_1^{1/2})\, M_1^{1/2} - 0.5\, M_1^{-3/2} s_1^{-1/2}\, B(1/2,\ 3/2)$$

$$\sum_{n=0} \left[\frac{(1/2)_n\,(3/2)_n}{(2)_n} \frac{(-1)^n}{n!} M_1^{-2n} \;\; {}_2F_1(1/4,\ n+3/2;\ n+2;\ q_c)\right]$$

(A.7) $$t_d = 0.4\,(1-s_1^{5/2})\,M_1^{1/2} - 0.5\,M_1^{-3/2}\,s_1^{3/2}\,B(1/2,\,3/2)$$

$$\sum_{n=0} [\frac{(1/2)_n\,(3/2)_n}{(2)_n}\,\frac{(-1)^n}{n!}\,M_1^{-2n}\,{}_2F_1(-3/4,n+3/2;n+2;q_c)]$$

$$q_c = -\,M_1^{-2}\,s_1^{-1}$$

(A.8) $$t_e = 0.5\,M_1^{1/2}\,[4 + \frac{\Gamma(1/2)\,\Gamma(-1/4)}{\Gamma(1/4)}\,[\sum_{n=0}\frac{(-M_1^{-2})^n}{n!}\,\frac{(1/2)_n\,(-1/4)_n}{(1/4)_n}$$

$${}_2F_1(-1/2,\,n-1/4;\,n+1/4;\,s_1^2)]]$$

(A.9) $$t_f = 0.5\,M_1^{1/2}\,[0.4 + \frac{\Gamma(1/2)\,\Gamma(-5/4)}{(-3/4)_n}\,[\sum_{n=0}\frac{(1/2)_n\,(-5/4)_n}{(-3/4)_n}\,\frac{(-M_1^{-2})^n}{n!}$$

$${}_2F_1(-1/2,\,n-5/4;\,n-3/4;\,s_1^2)]$$

(A.10) $$t_g = 0.5\,M_2^{1/2}[s_2^{3/2}\,B(3/2,\,1/4)\,{}_2F_1(-1/2,\,1/4;\,7/4;\,s_2^2)$$

$$-\,s_2^{7/2}\,M_2\,B(2,3/4)\,{}_2F_1(1/2,\,3/4;\,11/4;\,-\,s_2^2/M_2^2)]$$

(A.11) $$t_i = 0.5[M_2^{-7/2}\,s_2^{-3/2}\,B(3/2,3/2)\,{}_2F_1(3/4,3/2;\,3;\,-\,(M_2 s_2)^{-2}$$

$$-\,[(4/7)\,M_2^{-2}\,q_i^{7/4}\,F(7/4,9/4;11/4;q_i) - s_2^{7/2}\,{}_2F_1(7/4,9/4;11/4;s_2^2)$$

$$-\,(4/3)\,s_2^2\,q_i^{3/4}\,F(3/4,5/4;7/4;q_i) - s_2^{3/2}\,{}_2F_1(3/4,5/4;9/4;s_2^2)]]$$

$$q_i = c_2^2/(c_2^2 + v^2)$$

$$(A.12) \quad t_k = (1/3)\, M_2^{-3/2}\, s_2^2 - 0.5\, M_2^{-1/2} \Big[\sum_{n=2} \frac{(-\frac{1}{2})_n}{n!} \frac{M_2^{-2}}{(n-5/4)}\, {}_2F_1(3/4,1;\, n-1/4;\, s_2^2)$$

$$- \sum_{n=2} \frac{(1/2)_n}{n!} (-M^{-2})^n \Big[\frac{1}{n-5/4} - \frac{s_2^2}{n-1/4}\Big]\Big]$$

where

$$M_i = c_i/V \qquad\qquad i=1,2\ .$$
$$s_i^2 = 1 - (V/c_i)^2$$

The above results are obtained either by standard integration formulae or when necessary, by expanding the integrands and integrating term by term. The integration intervals are naturally divided by the presence of the branch points [15].

Depending on the values of (V/c_i), all series can be cast into rapdily converging forms by proper analytic continuation. The numerical value of the hypergeometric functions can also be found easily by evaluating the already existing sharp inequalities for lower and upper bounds. The relevant formulae for these inequalities and the necessary analytic continuation identies can be found in standard references.

REFERENCES

[1] Eringen, A.C., "Linear Theory of Nonlocal Elasticity and Dispersion of Plane Waves," Int. J. Engng. Sci., 10, 425-435, 1972.

[2] Yoffe, E.H., "The Moving Griffith Crack," Phil. Mag., 42, 739, 1951.

[3] Craggs, J.W., "On the Propagation of Crack in an Elastic-Brittle Solid," J. Mech. Phys. Solids, 8, 66, 1960.

[4] Broberg, K.B., "The Propagation of a Brittle Crack," Arhiv för Fysik, 18, 159, 1960.

[5] Baker, B.R., "Dynamic Stresses Created by a Moving Crack," J. Appl. Mech., 29, 449, 1962.

[6] Rose, L.R.F., "Recent Theoretical and Experimental Results on Fast Brittle Fracture," Int. J. of Fracture, 12, 799, 1976.

[7] Mott, N.F., "Brittle Fracture in Mild Steel Plates," Engineering, 165, 16, 1948.

[8] Roberts, D.K., and A.A. Wells, "The Velocity of Brittle Fracture," Engineering, 178, 820, 1954.

[9] Freund, L.B., "Dynamic Crack Propagation," in The Mechanics of Fracture, ed. by F. Erdogan, ASME/AMD - Vol. 19, 104, 1976.

[10] Lawn, B.R. and T.R. Wilshaw, Fracture of Brittle Solids, Cambridge University Press, London, 1975.

[11] Eringen, A.C., C.G. Speziale and B.S. Kim, "Crack Tip Problem in Nonlocal Elasticity," J. Mech. Phys. Solids, 25, 339, 1977.

[12] Ari, N. and A.C. Eringen, "Nonlocal Stress Field at Griffith Crack," submitted for publication to the Q. Jn. Mech. Appl. Math., 1981.

[13] Eringen, A.C., "Screw Dislocation in Nonlocal Elasticity," Jn.Phys. D: Appl. Phys., 10, 671, 1977.

[14] Eringen, A.C., "On the Nature of Boundary Conditions for Crack Tip Stress," 1980. Submitted for publication to Arch. Mech. (Poland).

[15] Ari, N., "High Field Gradient Phenomena in Nonlocal Media," Ph.D. Thesis, Princeton University, Dept. of Civil Eng., 1981.

[16] Sih, G.C. and Y.M. Chen, in Elastodynamic Crack Problems, ed. by G.C. Sih, Noordhoff, Int. Pub. Co., 1977.

[17] Eringen, A.C., "State of the Stress in the Neighborhood of a Sharp Crack Tip," ARO Report 77-1, 22nd Conf. of Army Mathematicians, Watenvliet, NY, 1976.

[18] Radon, C.J. and N.P. Fitzpatrick, "Deformation of PMMA at High Rates of Strain," in Dynamic Crack Propagation, ed. by G.C. Sih, Noordhoff Pub. Co., 227, 1972.

[19] Erdogan, F., in Fracture, II, ed. by H. Liebowitz, Academic Press, New New York, 1968.

[20] Freund, B.L., in Dynamic Crack Propagation, ed. by G.C. Sih, Noordhoff Pub. Co., 553, 1972.

[21] Eftis, J. and J.M. Krafft, Trans. ASME, 87, 257, 1965.

[22] Krafft, J.M. and A.M. Sullivan, ASM Trans. 56, 160, 1963.

[23] Williams, J.G., J.C. Radon and C.E. Turner, J. Poly. Eng. Sci., 130, 1968.

[24] Shmuely, M. and Z.S. Alterman, "Crack Propagation Abalysis by Finite Differences, " J. Applied Mech., 40, 902, 1973.

[25] Anthony, S.R., Chubb. J.P. and Congleton, J., "The Crack-Branching Velocity," Phil. Mag., 22, 1201, 1970.

Part 1 - Government
Administrative and Liaison Activities

Office of Naval Research
Department of the Navy
Arlington, Virginia 22217
Attn: Code 474 (2)
Code 471
Code 200

Director
Office of Naval Research
Eastern/Central Regional Office
666 Summer Street
Boston, Massachusetts 02210

Director
Office of Naval Research
Branch Office
536 South Clark Street
Chicago, Illinois 60605

Director
Office of Naval Research
New York Area Office
715 Broadway - 5th Floor
New York, New York 10003

Director
Office of Naval Research
Western Regional Office
1030 East Green Street
Pasadena, California 91106

Naval Research Laboratory (6)
Code 2627
Washington, D.C. 20375

Defense Technical Information Center (12)
Cameron Station
Alexandria, Virginia 22314

Navy

Undersea Explosion Research Division
Naval Ship Research and Development Center
Norfolk Naval Shipyard
Portsmouth, Virginia 23709
Attn: Dr. E. Palmer, Code 177

Navy (Con't.)

Naval Research Laboratory
Washington, D.C. 20375
Attn: Code 8400
8410
8430
8440
6300
6390
6380

David W. Taylor Naval Ship Research and Development Center
Annapolis, Maryland 21402
Attn: Code 2740
28
281

Naval Weapons Center
China Lake, California 93555
Attn: Code 4062
4520

Commanding Officer
Naval Civil Engineering Laboratory
Code L31
Port Hueneme, California 93041

Naval Surface Weapons Center
White Oak
Silver Spring, Maryland 20910
Attn: Code R-10
G-402
K-82

Technical Director
Naval Ocean Systems Center
San Diego, California 92152

Supervisor of Shipbuilding
U.S. Navy
Newport News, Virginia 23607

Navy Underwater Sound Reference Division
Naval Research Laboratory
P.O. Box 8337
Orlando, Florida 32806

Chief of Naval Operations
Department of the Navy
Washington, D.C. 20350
Attn: Code OP-098

Navy (Con't.)

Strategic Systems Project Office
Department of the Navy
Washington, D.C. 20376
Attn: NSP-200

Naval Air Systems Command
Department of the Navy
Washington, D.C. 20361
Attn: Code 5302 (Aerospace and Structures)
604 (Technical Library)
320B (Structures)

Naval Air Development Center
Warminster, Pennsylvania 18974
Attn: Aerospace Mechanics
Code 606

U.S. Naval Academy
Engineering Department
Annapolis, Maryland 21402

Naval Facilities Engineering Command
200 Stovall Street
Alexandria, Virginia 22332
Attn: Code 03 (Research and Development)
04B
045
14114 (Technical Library)

Naval Sea Systems Command
Department of the Navy
Washington, D.C. 20362
Attn: Code 05H
312
322
323
05R
32R

Navy (Con't.)

Commander and Director
David W. Taylor Naval Ship
Research and Development Center
Bethesda, Maryland 20084
Attn: Code 042
17
172
173
174
1800
1844
012.2
1900
1901
1945
1960
1962

Naval Underwater Systems Center
Newport, Rhode Island 02840
Attn: Bruce Sandman, Code 3634

Naval Surface Weapons Center
Dahlgren Laboratory
Dahlgren, Virginia 22448
Attn: Code G04
G20

Technical Director
Mare Island Naval Shipyard
Vallejo, California 94592

U.S. Naval Postgraduate School
Library
Code 0384
Monterey, California 93940

Webb Institute of Naval Architecture
Attn: Librarian
Crescent Beach Road, Glen Cove
Long Island, New York 11542

Army

Commanding Officer (2)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709
Attn: Mr. J. J. Murray, CRD-AA-IP

474:NP:716:lab
78u474-619

Army (Con't.)

Watervliet Arsenal
MAGGS Research Center
Watervliet, New York 12189
Attn: Director of Research

U.S. Army Materials and Mechanics
Research Center
Watertown, Massachusetts 02172
Attn: Dr. R. Shea, DRXMR-T

U.S. Army Missile Research and
Development Center
Redstone Scientific Information
Center
Chief, Document Section
Redstone Arsenal, Alabama 35809

Army Research and Development
Center
Fort Belvoir, Virginia 22060

NASA

National Aeronautics and Space
Administration
Structures Research Division
Langley Research Center
Langley Station
Hampton, Virginia 23365

National Aeronautics and Space
Administration
Associate Administrator for Advanced
Research and Technology
Washington, D.C. 20546

Air Force

Wright-Patterson Air Force Base
Dayton, Ohio 45433
Attn: AFFDL (FB)
(FBR)
(FBE)
(FBS)
AFML (MBM)

Chief Applied Mechanics Group
U.S. Air Force Institute of Technology
Wright-Patterson Air Force Base
Dayton, Ohio 45433

Air Force (Con't.)

Chief, Civil Engineering Branch
WLRC, Research Division
Air Force Weapons Laboratory
Kirtland Air Force Base
Albuquerque, New Mexico 87117

Air Force Office of Scientific Research
Bolling Air Force Base
Washington, D.C. 20332
Attn: Mechanics Division

Department of the Air Force
Air University Library
Maxwell Air Force Base
Montgomery, Alabama 36112

Other Government Activities

Commandant
Chief, Testing and Development Division
U.S. Coast Guard
1300 E Street, NW.
Washington, D.C. 20226

Technical Director
Marine Corps Development
and Education Command
Quantico, Virginia 22134

Director Defense Research
and Engineering
Technical Library
Room 3C128
The Pentagon
Washington, D.C. 20301

Dr. M. Gaus
National Science Foundation
Environmental Research Division
Washington, D.C. 20550

Library of Congress
Science and Technology Division
Washington, D.C. 20540

Director
Defense Nuclear Agency
Washington, D.C. 20305
Attn: SPSS

Other Government Activities (Con't)

Mr. Jerome Persh
Staff Specialist for Materials
and Structures
OUSDR&E, The Pentagon
Room 3D1089
Washington, D.C. 20301

Chief, Airframe and Equipment Branch
FS-120
Office of Flight Standards
Federal Aviation Agency
Washington, D.C. 20553

National Academy of Sciences
National Research Council
Ship Hull Research Committee
2101 Constitution Avenue
Washington, D.C. 20418
Attn: Mr. A. R. Lytle

National Science Foundation
Engineering Mechanics Section
Division of Engineering
Washington, D.C. 20550

Picatinny Arsenal
Plastics Technical Evaluation Center
Attn: Technical Information Section
Dover, New Jersey 07801

Maritime Administration
Office of Maritime Technology
14th and Constitution Avenue, NW.
Washington, D.C. 20230

PART 2 - Contractors and Other Technical Collaborators

Universities

Dr. J. Tinsley Oden
University of Texas at Austin
345 Engineering Science Building
Austin, Texas 78712

Professor Julius Miklowitz
California Institute of Technology
Division of Engineering
and Applied Sciences
Pasadena, California 91109

Universities (Con't)

Dr. Harold Liebowitz, Dean
School of Engineering and
Applied Science
George Washington University
Washington, D.C. 20052

Professor Eli Sternberg
California Institute of Technology
Division of Engineering and
Applied Sciences
Pasadena, California 91109

Professor Paul M. Naghdi
University of California
Department of Mechanical Engineering
Berkeley, California 94720

Professor A. J. Durelli
Oakland University
School of Engineering
Rochester, Missouri 48063

Professor F. L. DiMaggio
Columbia University
Department of Civil Engineering
New York, New York 10027

Professor Norman Jones
The University of Liverpool
Department of Mechanical Engineering
P. O. Box 147
Brownlow Hill
Liverpool L69 3BX
England

Professor E. J. Skudrzyk
Pennsylvania State University
Applied Research Laboratory
Department of Physics
State College, Pennsylvania 16801

Professor J. Klosner
Polytechnic Institute of New York
Department of Mechanical and
Aerospace Engineering
333 Jay Street
Brooklyn, New York 11201

Professor R. A. Schapery
Texas A&M University
Department of Civil Engineering
College Station, Texas 77843

Universities (Con't.)

Professor Walter D. Pilkey
University of Virginia
Research Laboratories for the
Engineering Sciences and
Applied Sciences
Charlottesville, Virginia 22901

Professor K. D. Willmert
Clarkson College of Technology
Department of Mechanical Engineering
Potsdam, New York 13676

Dr. Walter E. Haisler
Texas A&M University
Aerospace Engineering Department
College Station, Texas 77843

Dr. Hussein A. Kamel
University of Arizona
Department of Aerospace and
Mechanical Engineering
Tucson, Arizona 85721

Dr. S. J. Fenves
Carnegie-Mellon University
Department of Civil Engineering
Schenley Park
Pittsburgh, Pennsylvania 15213

Dr. Ronald L. Huston
Department of Engineering Analysis
University of Cincinnati
Cincinnati, Ohio 45221

Professor G. C. M. Sih
Lehigh University
Institute of Fracture and
Solid Mechanics
Bethlehem, Pennsylvania 18015

Professor Albert S. Kobayashi
University of Washington
Department of Mechanical Engineering
Seattle, Washington 98105

Professor Daniel Frederick
Virginia Polytechnic Institute and
State University
Department of Engineering Mechanics
Blacksburg, Virginia 24061

Universities (Con't)

Professor A. C. Eringen
Princeton University
Department of Aerospace and
Mechanical Sciences
Princeton, New Jersey 08540

Professor E. H. Lee
Stanford University
Division of Engineering Mechanics
Stanford, California 94305

Professor Albert I. King
Wayne State University
Biomechanics Research Center
Detroit, Michigan 48202

Dr. V. R. Hodgson
Wayne State University
School of Medicine
Detroit, Michigan 48202

Dean B. A. Boley
Northwestern University
Department of Civil Engineering
Evanston, Illinois 60201

Professor P. G. Hodge, Jr.
University of Minnesota
Department of Aerospace Engineering
and Mechanics
Minneapolis, Minnesota 55455

Dr. D. C. Drucker
University of Illinois
Dean of Engineering
Urbana, Illinois 61801

Professor N. M. Newmark
University of Illinois
Department of Civil Engineering
Urbana, Illinois 61803

Professor E. Reissner
University of California, San Diego
Department of Applied Mechanics
La Jolla, California 92037

Professor William A. Nash
University of Massachusetts
Department of Mechanics and
Aerospace Engineering
Amherst, Massachusetts 01002

Universities (Con't)

Professor G. Herrmann
Stanford University
Department of Applied Mechanics
Stanford, California 94305

Professor J. D. Achenbach
Northwest University
Department of Civil Engineering
Evanston, Illinois 60201

Professor S. B. Dong
University of California
Department of Mechanics
Los Angeles, California 90024

Professor Burt Paul
University of Pennsylvania
Towne School of Civil and
Mechanical Engineering
Philadelphia, Pennsylvania 19104

Professor H. W. Liu
Syracuse University
Department of Chemical Engineering
and Metallurgy
Syracuse, New York 13210

Professor S. Bodner
Technion R&D Foundation
Haifa, Israel

Professor Werner Goldsmith
University of California
Department of Mechanical Engineering
Berkeley, California 94720

Professor R. S. Rivlin
Lehigh University
Center for the Application
of Mathematics
Bethlehem, Pennsylvania 18015

Professor F. A. Cozzarelli
State University of New York at
Buffalo
Division of Interdisciplinary Studies
Karr Parker Engineering Building
Chemistry Road
Buffalo, New York 14214

Universities (Con't)

Professor Joseph L. Rose
Drexel University
Department of Mechanical Engineering
and Mechanics
Philadelphia, Pennsylvania 19104

Professor B. K. Donaldson
University of Maryland
Aerospace Engineering Department
College Park, Maryland 20742

Professor Joseph A. Clark
Catholic University of America
Department of Mechanical Engineering
Washington, D.C. 20064

Dr. Samuel B. Batdorf
University of California
School of Engineering
and Applied Science
Los Angeles, California 90024

Professor Isaac Fried
Boston University
Department of Mathematics
Boston, Massachusetts 02215

Professor E. Krempl
Rensselaer Polytechnic Institute
Division of Engineering
Engineering Mechanics
Troy, New York 12181

Dr. Jack R. Vinson
University of Delaware
Department of Mechanical and Aerospace
Engineering and the Center for
Composite Materials
Newark, Delaware 19711

Dr. J. Duffy
Brown University
Division of Engineering
Providence, Rhode Island 02912

Dr. J. L. Swedlow
Carnegie-Mellon University
Department of Mechanical Engineering
Pittsburgh, Pennsylvania 15213

Universities (Con't)

Dr. V. K. Varadan
Ohio State University Research Foundation
Department of Engineering Mechanics
Columbus, Ohio 43210

Dr. Z. Hashin
University of Pennsylvania
Department of Metallurgy and Materials Science
College of Engineering and Applied Science
Philadelphia, Pennsylvania 19104

Dr. Jackson C. S. Yang
University of Maryland
Department of Mechanical Engineering
College Park, Maryland 20742

Professor T. Y. Chang
University of Akron
Department of Civil Engineering
Akron, Ohio 44325

Professor Charles W. Bert
University of Oklahoma
School of Aerospace, Mechanical, and Nuclear Engineering
Norman, Oklahoma 73019

Professor Satya N. Atluri
Georgia Institute of Technology
School of Engineering and Mechanics
Atlanta, Georgia 30332

Professor Graham F. Carey
University of Texas at Austin
Department of Aerospace Engineering and Engineering Mechanics
Austin, Texas 78712

Dr. S. S. Wang
University of Illinois
Department of Theoretical and Applied Mechanics
Urbana, Illinois 61801

Professor J. F. Abel
Cornell University
Department of Theoretical and Applied Mechanics
Ithaca, New York 14853

Universities (Con't)

Professor V. H. Neubert
Pennsylvania State University
Department of Engineering Science and Mechanics
University Park, Pennsylvania 16802

Professor A. W. Leissa
Ohio State University
Department of Engineering Mechanics
Columbus, Ohio 43212

Professor C. A. Brebbia
University of California, Irvine
Department of Civil Engineering
School of Engineering
Irvine, California 92717

Dr. George T. Hahn
Vanderbilt University
Mechanical Engineering and Materials Science
Nashville, Tennessee 37235

Dean Richard H. Gallagher
University of Arizona
College of Engineering
Tucson, Arizona 85721

Professor E. F. Rybicki
The University of Tulsa
Department of Mechanical Engineering
Tulsa, Oklahoma 74104

Dr. R. Haftka
Illinois Institute of Technology
Department of Mechanics and Mechanical and Aerospace Engineering
Chicago, Illinois 60616

Professor J. G. de Oliveira
Massachusetts Institute of Technology
Department of Ocean Engineering
77 Massachusetts Avenue
Cambridge, Massachusetts 02139

Dr. Bernard W. Shaffer
Polytechnic Institute of New York
Route 110
Farmingdale, New York 11735

Industry and Research Institutes

Dr. Norman Hobbs
Kaman AviDyne
Division of Kaman
Sciences Corporation
Burlington, Massachusetts 01803

Argonne National Laboratory
Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. M. C. Junger
Cambridge Acoustical Associates
54 Rindge Avenue Extension
Cambridge, Massachusetts 02140

Mr. J. H. Torrance
General Dynamics Corporation
Electric Boat Division
Groton, Connecticut 06340

Dr. J. E. Greenspon
J. G. Engineering Research Associates
3831 Menlo Drive
Baltimore, Maryland 21215

Newport News Shipbuilding and
Dry Dock Company
Library
Newport News, Virginia 23607

Dr. W. F. Bozich
McDonnell Douglas Corporation
5301 Bolsa Avenue
Huntington Beach, California 92647

Dr. H. N. Abramson
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Industry and Research Institutes (Con't)

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedalean Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. H. D. Hibbitt
Hibbitt & Karlsson, Inc.
132 George M. Cohan Boulevard
Providence, Rhode Island 02903

Dr. R. D. Mindlin
89 Deer Hill Drive
Ridgefield, Connecticut 06877

Industry and Research Institutes (Con't)

Dr. Richard E. Dame
Mega Engineering
11961 Tech Road
Silver Spring, Maryland 20904

Mr. G. M. Stanley
Lockheed Palo Alto Research
Laboratory
3251 Hanover Street
Palo Alto, California 94304

Mr. R. L. Cloud
Robert L. Cloud Associates, Inc.
2972 Adeline Street
Berkeley, California 94703

DATE
ILMED
-8